



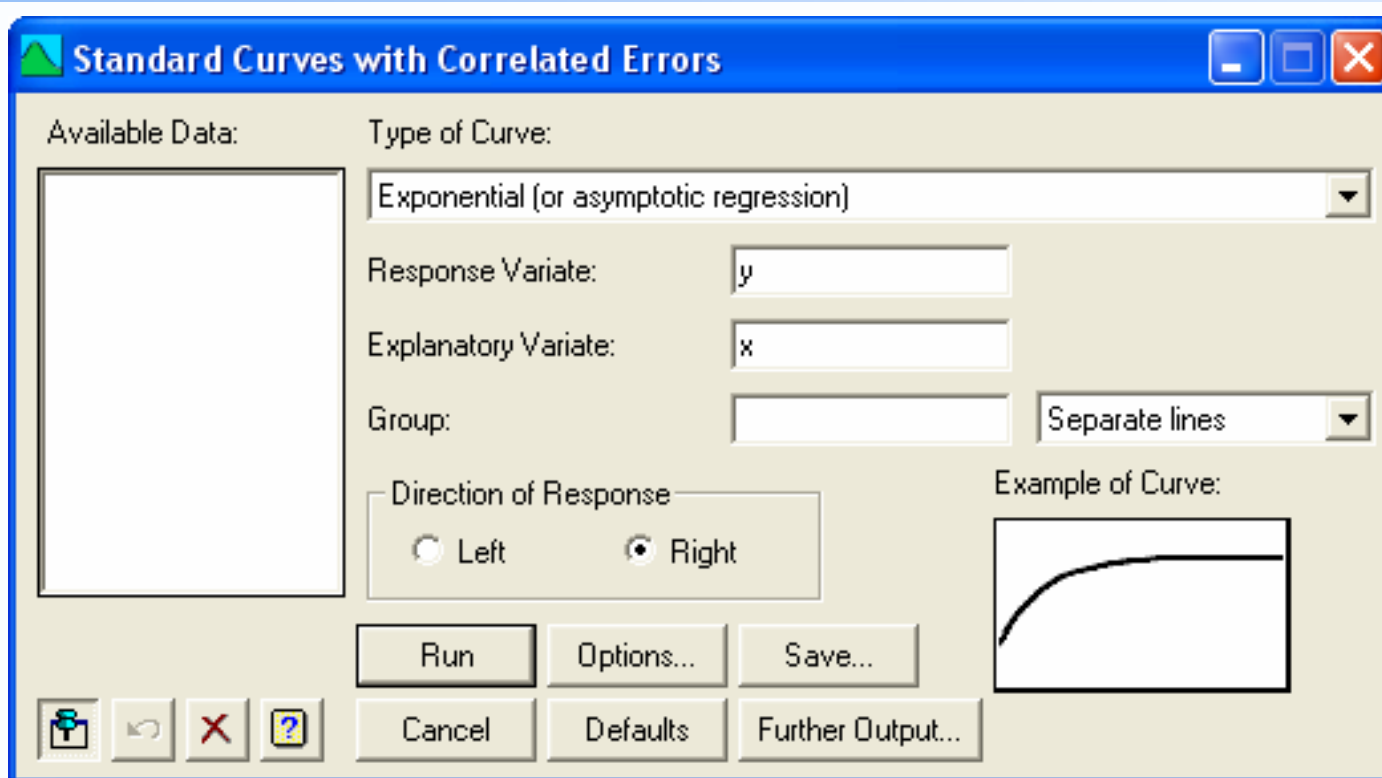
Fitting curves and regressions with repeated measurements

Roger Payne

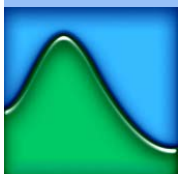
VSN International, 5 The Waterhouse,
Waterhouse Street, Hemel Hempstead, UK

Wageningen, 18th June 2008

Curves with correlated errors



- *Standard Curves with Correlated Errors* menu
 - very similar to the *Standard Curves* menu
 - but automatically fits a correlation model along the x-direction
 - correlation between points x_1 and x_2 is $\varphi^{x_1 - x_2}$
 - AR1 structure (equally-spaced x's) or power-distance model



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Curves with correlated errors

Standard Curve with Correlated Errors Options

Display

- Model
- Summary
- E-probability
- Correlations
- Phi
- Estimates
- Fitted Values
- Accumulated
- Monitoring Estimation of Phi

Graphics

- Plot Residuals
- Plot Fitted Model
- Plot Likelihood for Phi

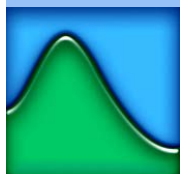
Estimate Constant Term Constrain Origin

Weights:

Maximum iterations to estimate phi:

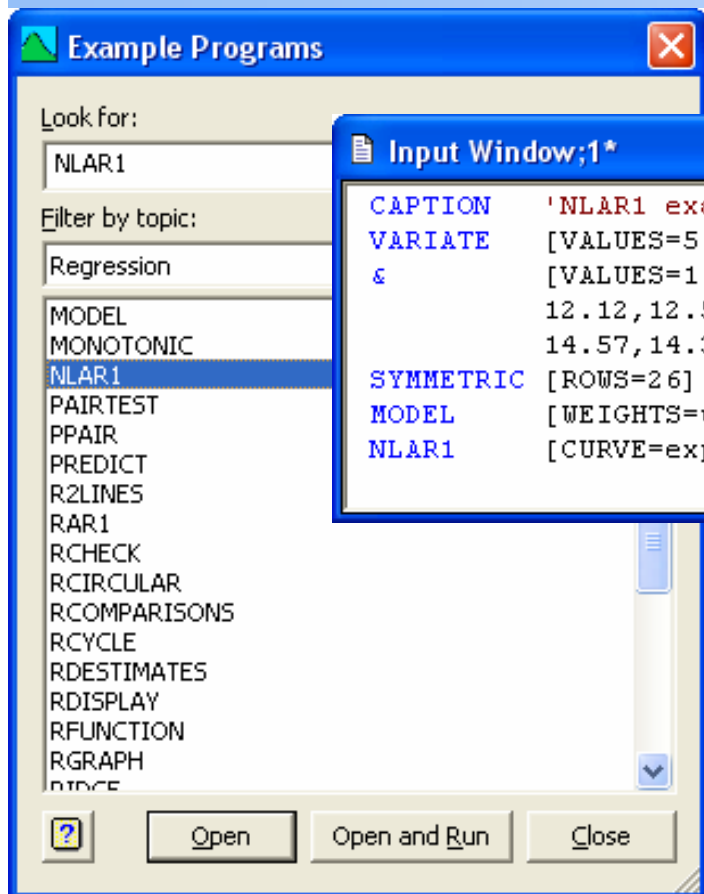
Buttons: [Close] [Help] [OK] [Cancel] [Defaults]

- *Standard Curves with Correlated Errors Options*
 - very similar to the *Standard Curves Options* menu
 - but allows for weights
 - and extra output options to display/monitor φ



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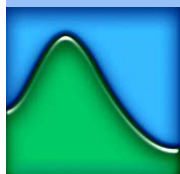
Example – growth curve



```
Input Window;1*  
CAPTION 'NLAR1 example'; STYLE=meta  
VARIATE [VALUES=5...30] x  
& [VALUES=1.30,3.55,5.13,6.48,7.85,8.96,9.84,10.91,11.29,11.  
12.12,12.55,12.70,13.14,13.47,13.78,14.01,14.11,14.55,14.7  
14.57,14.30,14.67,14.68,15.03,15.00] y  
SYMMETRIC [ROWS=26] wt  
MODEL [WEIGHTS=wt] y  
NLAR1 [CURVE=exponential] x
```

Row	x	y
1	5	1.3
2	6	3.55
3	7	5.13
4	8	6.48
5	9	7.85
6	10	8.96
7	11	9.84
8	12	10.91
9	13	11.29
10	14	11.76
11	15	12.12
12	16	12.55
13	17	12.7
14	18	13.14
15	19	13.47
16	20	13.78
17	21	14.01
18	22	14.11
19	23	14.55
20	24	14.71

- example of procedure NLAR1 (which fits model)
 - access using the *Example Programs* menu
 - or from spreadsheet (stored earlier)
- y is related to x by an exponential model



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Output

```
Nonlinear regression analysis
=====
```

```
Response variate: y
Weight matrix: _wgtmat based on power-distance correlation model
Explanatory: x
Fitted Curve: A + B*(R**X)
Constraints: R < 1
```

← model (with estimated weight matrix based on φ)

```
Summary of analysis
-----
```

Source	d.f.	s.s.	m.s.	v.r.	F pr.
Regression	2	196.4891	98.24454	3132.82	<.001
Residual	23	0.7213	0.03136		
Total	25	197.2104	7.88841		

← summary etc (as usual)

```
Percentage variance accounted for 99.6
Standard error of observations is estimated to be 0.177.
```

```
* MESSAGE: the following units have large standardized residuals.
```

Unit	Response	Residual
8	10.910	2.46
13	12.700	-2.37
22	14.300	-2.22

```
* MESSAGE: the residuals do not appear to be random;
for example, fitted values in the range 11.793 to 14.177
are consistently larger than observed values
and fitted values in the range 7.808 to 11.226
are consistently smaller than observed values.
```

← residuals non random because of correlation structure (ignore!)

```
* MESSAGE: the following units have high leverage.
```

Unit	Response	Leverage
1	1.300	0.50
2	3.550	0.28

..

Output

Estimates of parameters

```
-----  
Parameter      estimate      s.e.  
R              0.85432     0.00282  
B             -30.166     0.581  
A              15.1216    0.0732
```

← curve parameters

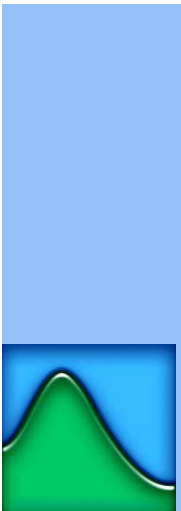
Correlation parameter estimate

```
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Phi: 0.4008
```

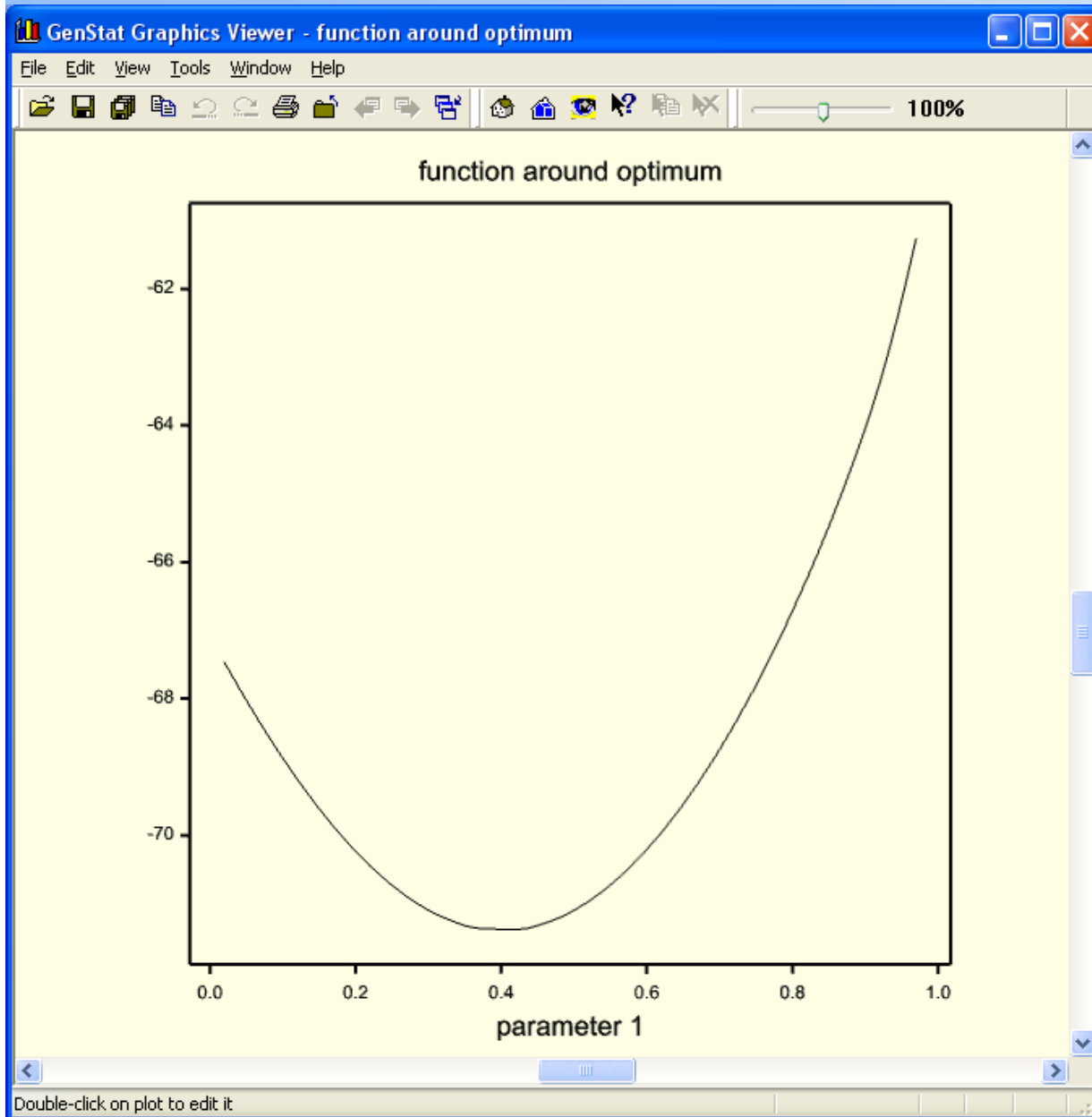
```
Test for phi non-zero: chi-square 4.313 on 1 d.f., probability 0.038
```

← estimate for φ (and test for φ non-zero)

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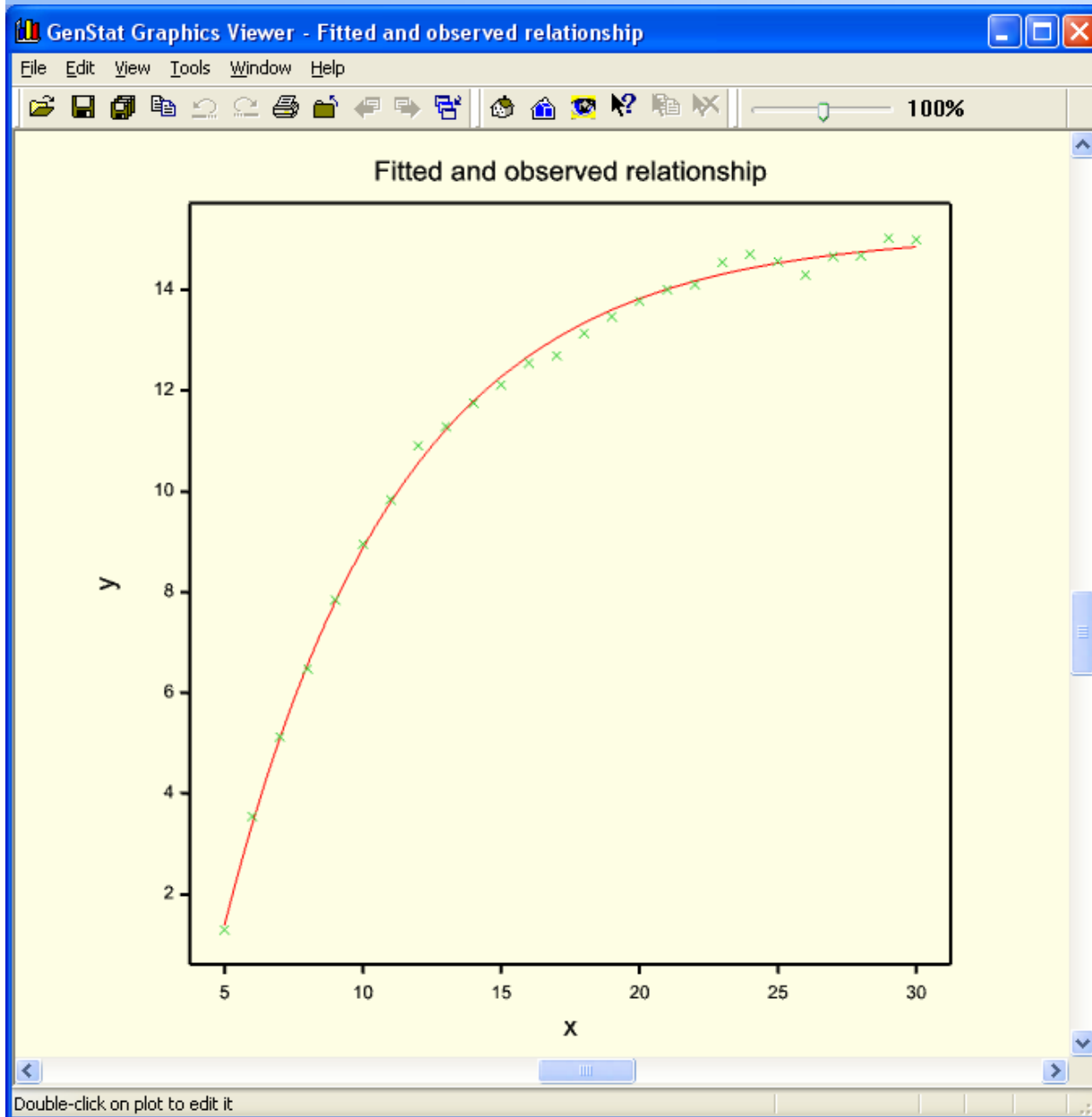
Curves with correlated errors



- likelihood for φ
 - plotted by procedure MIN1DIMENSION, which estimates φ (by maximum likelihood)

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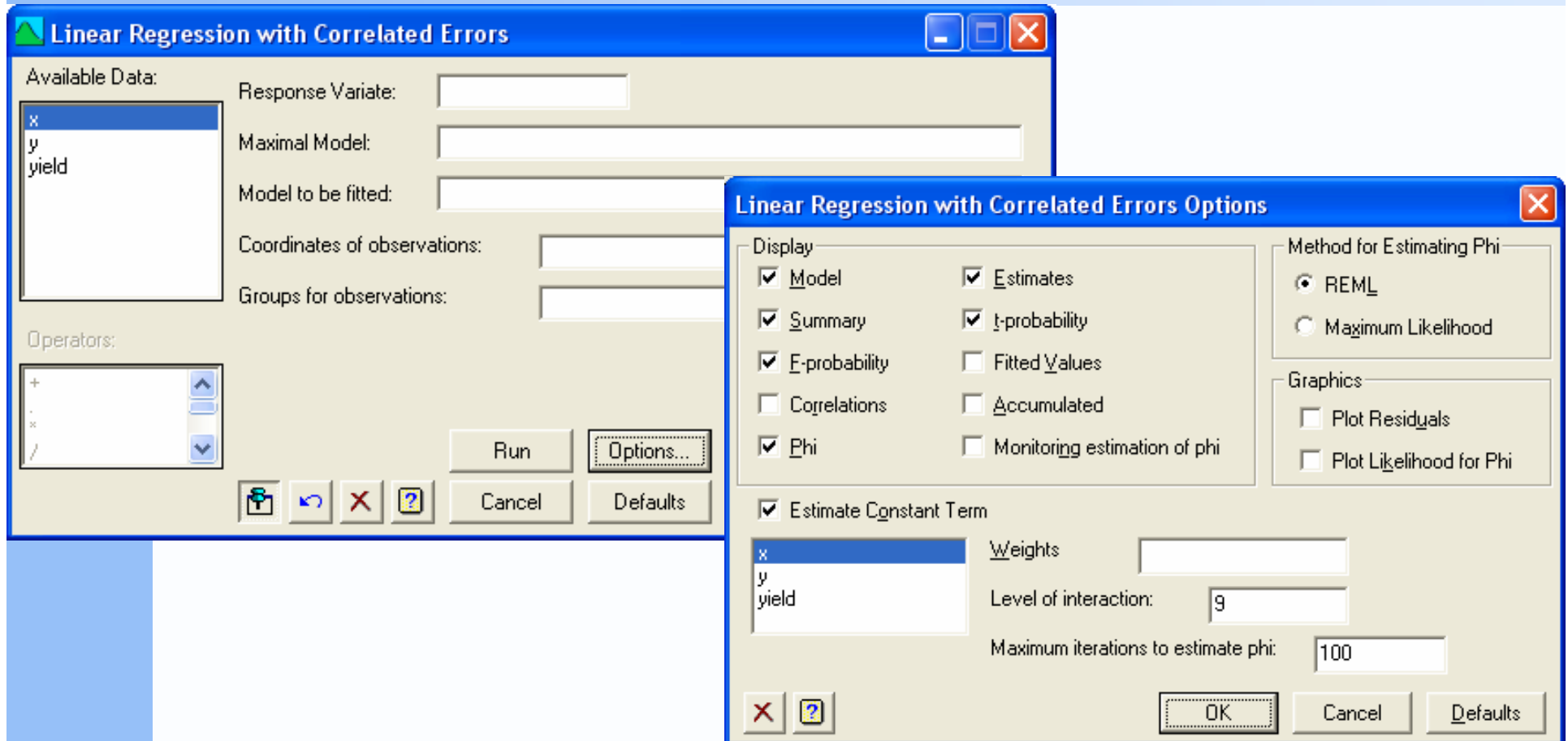
Curves with correlated errors



- fitted model
 - notice the "non random" pattern of the residuals (caused by the correlation structure)

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Linear regression with correlated errors



- *Linear Regression with Correlated Errors* menu
 - similar to the *Standard Curves with Correlated Errors* menu
 - but ϕ can be fitted by REML (as well as maximum likelihood)

